A warning about the use of reduced models

of synchronous generators

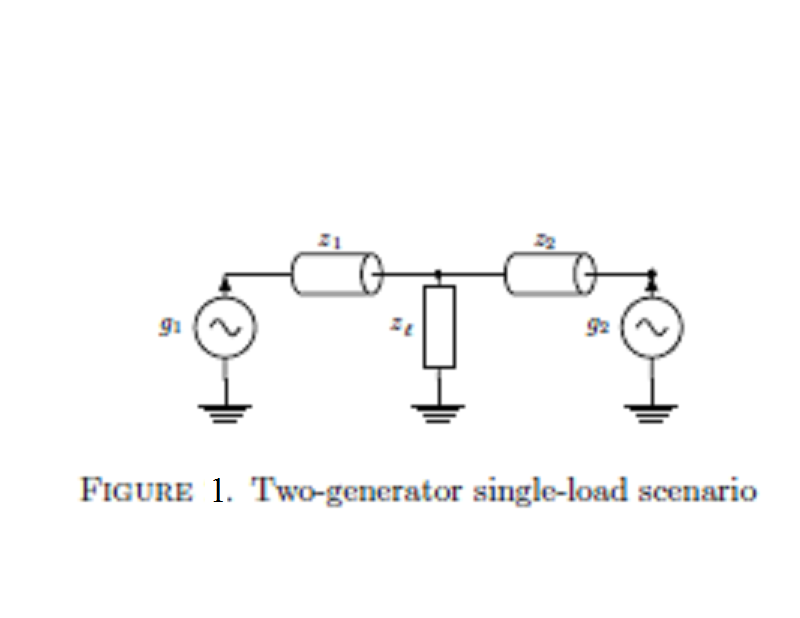
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1. INTRODUCTION

The mathematical model of a round rotor synchronous generator (SG) (without damper windings), when neglecting the nonlinear effects in the iron core, is a system of five bilinear differential equations, see Section II for the details. If we add various control loops (such as power system stabilizers) then the order of the model increases. Many such SGs are coupled together in the electricity grid, connected via transmission lines and transformers, and there are of course also loads. This makes the electric grid such a complicated nonlinear and time-varying system that any attempt to prove its stability analytically is hopeless. Stability analysis is usually done by simulation, or analytically on simplified models, in which the SGs are connected in a simple network and each SG is represented by reduced order equations, see for instance Doerfler and Bullo [?,?], Porco, Doerfler and Bullo [?], ……. The reduced model of a SG is often obtained by treating the stator currents as fast variables, thus eliminating them from the state variables via the singular perturbation approach (see, for instance, Khalil [?]) and keeping only the rotor angle, the rotor angular velocity and the rotor current as relevant state variables, see for instance Kundur ??? [?], ….

In this paper we want to give a warning about using reduced order models for stability analysis. We consider a very simple system consisting of two identical SGs connected at the two ends of a power line represented only by its inductance and resistance. There is a resistive load connected at the middle of this power line, as shown in Figure 1, which is similar to Figure 2 in Celiska and Tabuada [?].



For simplicity, we consider the rotor currents to be constant (this is equivalent to considering permanent magnet generators). Thus, the full model of each SG is of order 4 (instead of 5). We also consider the representation of these SGs by their reduced order models, which are of order two. We show that, for reasonable choices of the parameters, the following situation may occur:

The system built with reduced order models of the SGs is locally stable around one equilibrium point (and unstable around another), simulations seem to indicate that it is even almost globally asymptotically stable, but the system built with the full (4th order) models of the SGs is not even locally stable around any equilibrium point. Since this phenomenon can occur even for the very simple network shown above, it is of course even more likely to occur for more complex networks.

Along the way, we prove some minor facts about the network shown in Figure 1. Let us denote by the angle difference between the rotors of the two SGs. We show that for any equilibrium point, either (modulo by ) or (modulo by ) and the states of the two SGs are equal. Under a certain natural assumption on the generator parameters, there is a unique equilibrium point for (modulo by ) and a unique equilibrium point with (modulo by ).

1. THE MODEL OF A SYNCHRONOUS MACHINE
2. THE MODEL OF TWO COUPLED SYNCHRONOUS MACHINES
3. STABILITY ANALYSIS FOR TWO COUPLED MACHINES

**The full model:**

As we showed, a model for two generators is:

Where:

,

**The reduced model:**

We can write the model in the following way:

Where

, ,

We assume that is very small which means that is much faster than, so it get to it equilibrium point (which we assume that exist, and attractive throw all paths of ) almost instantly with comparing to .

So we will calculate the root of (it is linear system)

and we will use it in order to solve the three order system:

Although this system is only 3rd order system, this nonlinear system is very complex. In order to simplify the dynamic of the reduced model, we will substitute also at the right hand side of and we will show the dynamic for . [See Khalil Non Linear Systems 3rd edition pages 423 – 460].

And now

**Specific case: Almost 10 KW generator parameters.**

Let's examinant case of 10 KW generator parameters, where we will replace the 10 KW inductor value ( 2.2 [mH]) with 0.1 [mH] inductor.

The system parameters:

First, let's find the system equilibrium points:

As we showed, the system equilibrium points must satisfy:

**1. We will start with:**

We will find with the cubic equation from page 12 at the notes:

Where

Solving this equation give the following solution:

The only real solution gives

We will calculate with the dynamics of the third line:

We will calculate with the dynamics of the second line:

In order to validate those results, let's calculate

This is MATLAB numerical error.

**2. Now, :**

We will get the same results as at the previous section, but with

Solving:

Gives:

Now,

In order to validate those results, let's calculate

This is MATLAB numerical error.

Now we will calculate the Jacobian of this system:

Where:

We will substitute our parameters and our equilibrium points into and calculate (numerically) its eigenvalues:

**For :**

We get:

This shows that this equilibrium point is not stable.

**For :**

We get:

This shows that this equilibrium point is not stable.

**The reduced model:**

Let's validate that the equilibrium point which we found (for ) is also equilibrium point for the reduced model.

This is very small number, but it is not small as the value which we got for the full model , because we neglect the inductors when we calculate the equilibrium point of the fast dynamics.

Now, in order to investigate the stability of the system we will calculate the Jacobian of this system, at the equilibrium point:

It's easy to see that at the equilibrium point, the third column will be zero, so we will have zero eigenvalue, which means that our reduction is not clear enough.

**More accurate model:**

In order to have more accurate reduction, let's neglect factors with , but we will still have factors with :

And now:

We will substitute the equilibrium point and calculate the eigenvalue:

It shows that although the system is not stable, the model reduction is stable.

We will show simulation of system with these parameters set which starts from arbitrary initial point:

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